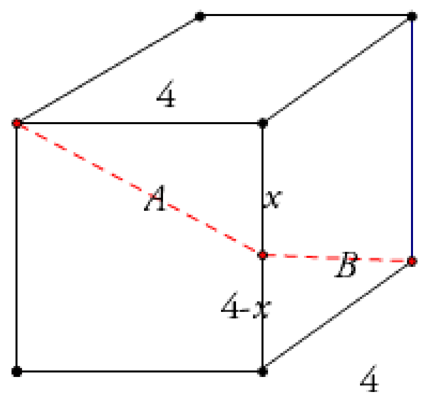
Chapter 3 Lab

10. We use optimization.
In order for the ant to walk from one corner to the opposite, it must walk through a minimum of two faces. Let the length of the walk of the ant on face one be A and the walk on face two be B. This is shown in the diagram to the right. We can express A and B in terms of x:



$$A = \sqrt{4^2 + x^2}$$
$$B = \sqrt{4^2 + (4 - x)^2}$$

$$A = \sqrt{16 + x^2}$$
$$B = \sqrt{x^2 - 8x + 32}$$

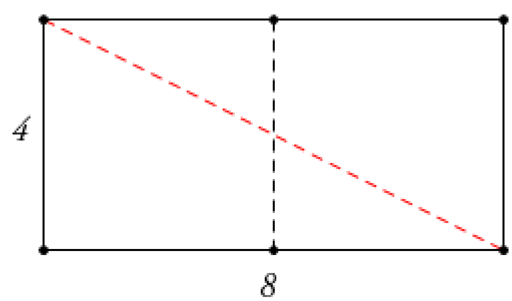
If Z is the length of the path, then $Z = A + B = \sqrt{16 + x^2} + \sqrt{x^2 - 8x + 32}$. We set the derivative - $\frac{x-4}{\sqrt{x^2-8x+32}} + \frac{x}{\sqrt{x^2+16}} = 0$.

Solving for x:

$$(x-4)\sqrt{x^2+16} = -x\left(\sqrt{x^2 - 8x + 32}\right)$$
$$(x-4)^2(x^2+16) = x^2(x^2 - 8x + 32)$$
$$x^4 - 8x^3 + 32x^2 - 128x + 256 = x^4 - 8x^3 + 32x^2$$
$$-128x + 256 = 0$$
$$128x = 256$$
$$x = 2$$

We now plug $x = 2$ back into our original function and have ≈ 8.94.

This problem may also be solved without calculus by making the two involved faces into a two dimensional rectangle with sides 8 and 4. We can now apply Pythagoras and we have the same answer.



14. The slope of the tangent line depends on the value of the first derivative. At $x = 0$, $f'(x) = 0$, and there is no slope. There is the least slope at this point. This point is (0,1).

We must now find the point on the graph where the slope of the tangent line is greatest, i.e. where $|f'(x)|$ is the greatest.

Taking the derivative of $f(x) = \frac{1}{1+x^2}$, we have $-\frac{2x}{(x^2+1)^2}$. In the graph of the derivative of the original function, we want the relative maximum. This can be done by taking the second derivative of the original function, i.e. $\frac{2(3x^2-1)}{(x^2+1)^3}$. We find the zeroes of this, which are $\pm\frac{\sqrt{3}}{3}$. These two points where the slope of the tangent line is maximized are $\left(\frac{\sqrt{3}}{3}, \frac{3}{4}\right)$ and $\left(-\frac{\sqrt{3}}{3}, \frac{3}{4}\right)$.